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ABELIAN CALIBRATION fiELD THEORY ON U (1) LIE GROUP WITH LORENTZ INVARIANTS IN 2+1 DIMENSIONS WITH TOPOLOGICAL MASS GENERATION.

A. Alatorre [*1]
[*] Department of physics, University of Guadalajara.

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ABSTRACT
Main features and mechanism of a calibration abelian theory constructed on U(1) Lie group is presented. Given Lorentz invariants for which local invariance principle is satisfied, its respective action is built. Calibration field and its curvature are obtained from U(1) symmetries. Maxwell electromagnetic formalism has been developed for this model. Induced Chern-Simmons action is reduced to 2+1 dimensions and Maxwell equations rewrote. Both magnetic and electric field equations are solved from Maxwell theory, and obtained Klein Gordon massive wave field equations, which presented topological mass.

KEYWORDS: Calibration theory, topological field theory, topological mass, Chern-Simmons application, 2+1 field dimensions, quantum cosmology.

1. INTRODUCTION
This paper has the purpose to show the mechanism and steps to follow in order to obtain a topological mass generation theory in 2+1 dimensions. Many of these works have been done in the same branch of field theory in physics [1,2,3,4,5,6]. Topological mass, and in general topological order, is a subject for which many debates remain open. Field equations are obtained containing topological mass as the result of Chern-Simmons action induction. Following abelian field theory over U(1) Lie group, and defining adequate Lorenz invariance, and then Lorenz-Gauge invariants due to unitary group of dimension 1 symmetry, it is possible to construct a calibration action. This action may be reduced to a 2+1 dimensional reduction due to Chern-Simmons induction over abelian basic model. Therefore, Maxwell tensor equations and Chern-Simmons approach appears as main actors in this formulation. Action is rewritten in terms of Chern-Simmons action and it is taken to Maxwell formalism for electric and magnetic field. These equations might be solved obtaining Klein-Gordon equations as a result. It will be evident that integration constants kn for Klein-Gordon equations belong in their units to topological mass, and that, Klein Gordon equations themselves represent a typical case of mass wave propagation phenomena. Einstein tensorial and relativistic formalism is used. this article is not planned to be a full theory construction article, but a calibration theory with emphasis in topological mass generation. Therefore, many details are omitted, and other regular features casual to find in field theory development are not written here.

1.1 MOTIVATION
From [1,2,6] give us a hint on the mechanisms to obtain topological mass on low-dimensional relativistic space-time manifolds. Actual topological Gauge field theory uses plenty of mechanisms in order to obtain topological field theories with topological order properties such as topological mass, topological Hall fluids and topological space-time manifolds. A full topological theory has been made with these last elements, and it also includes hierarchy of coupling constants of topological order [4]. Also, calibration theory gives us an adequate formalism to obtain field actions. Understanding topological objects from a physical point of view, will increase our possibilities to develop experimental and accurate cosmological and field theories [4] It is important to theoretical physics advance to achieve further knowledge regarding to different types of topological mass generation in a wide variety of dimensions. It is usual to obtain them since Chern-Simmons formulations. The main purpose of study of topological mass is to contribute in the advance of topological field theory, and topological physical interpretations of nature, particle/field interactions and space-time.

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1.2 ANTECEDENTS

Topological mass generation, and in general, topological order, based on fluids and topological states, has been object of debate in latest years [8,9]. In one side, one has the apologetics for one of this two stands of discussion; Landau fractional states, and topological order. This discussion took place for the last 60 years. Many of its history and evolution is gathered in [7,10]. Symmetry and commutative groups are involved in this study. Field basic theory says that, Yang-Mills theory of mass gap is obtained when symmetry principle it is not satisfy in mass scalar fields.

1.3 ARTICLE STRUCTURE

This article is structured by following summary:
1.- Introduction.
2.- Defining Lorentz-Gauge invariance. 2.- Building calibration action on U(1).
3.- Action functional.
4.- Field expressions
5.- 2+1 Chern-Simmons dimension reduction.
6.- Maxwell and Chern-Simmons approximation.
7.- Chern-Simmons 2+1 action in Maxwell formalism.
8.- Klein Gordon output equations.
9.- Topological mass analysis.
10.- Summary.
11.- Conclusions
- References.

2. DEFINING GAUGE-LORENZ INVARIANCE.

Calibration field is A_μ and it is abelian. In electromagnetism, the Lorenz condition or Lorenz-Gauge condition is a partial Gauge fixing of the electromagnetic vector potential, which consists in $\partial_\mu A_\mu = 0$. And this last expression show us that potential field will follow Gauge transformation $A^\mu \longrightarrow A^\mu \partial^\mu f$ where f is a harmonic scalar function (that is, a scalar function satisfying $\partial_\mu \partial^\mu f = 0$).

2.1 LORENTZ INVARIANTS

There are two Lorenz invariants for field A_μ which are $F_{\mu\nu}F^{\mu\nu}$ and $e^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$. These two invariants satisfy the Gauge condition for Lorenz relativistic invariance. Setting these invariants into field equations, it allows us to obtain independent scalar fields invariant under Gauge-Lorenz transformations [6,15].

2.2 LORENTZ-GAUGE TRANSFORMATIONS.

Field A_μ change due to Lorentz transformation:

$$A_\mu \to A'_\mu(x) = A_\mu + \partial_\mu \alpha(x) \qquad (1)$$

Hence the Maxwell Lagrangian density is manifestly invariant under small gauge transformations since $\partial_\mu J^\mu = 0$ and $F'_{\mu\nu}$. For Maxwell theory, a Gauge field as $A_\mu(x)$ may be split through the Hodge decomposition into longitudinal and transverse parts:

$$A_\mu = A^L_\mu + A^T_\mu \;=\; \partial_\mu \theta + \eta^{\rho\sigma}\partial_\rho \xi_{\mu\sigma} \qquad (2)$$

and one considered field A_μ curvature given by:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad (3)$$

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where ∂_μ is the derivative with respect to the vector coordinate x while the matter current $J^\mu(x)$ is conserved: $\partial_\mu J_\mu = 0$ This latter continuity condition which directly results from the Maxwell electromagnetism equations:

$$\eta^{\nu\sigma}\eta^{\nu\rho}\partial\rho\, F\mu\nu = e^2 J^\sigma \qquad (4)$$

is in fact related the invariance of the theory under the abelian U (1) gauge transformations [12]. If we consider $j^\mu(x)$ as the current associated to dynamical matter fields which couple to the gauge-calibration field, and this conservation law is an expression of Noether's theorem [10].

We define abelian gauge transformations under Lorentz invariance thru smooth maps:

$$U : M \rightarrow U(1) : (t,x) \rightarrow U(t,x = e^{i\alpha(t,x)}) \qquad (5)$$

The parameter $\alpha(x)$ of the transformation depends on space-time coordinates. The transformation $U(t,x)$ acts on the connection $A_\mu(x)$ and its associated field strength curvature tensor $F_{\mu\nu}$ in the usual Gauge transformation theory [12]:

$$A'_\mu = UA_\mu A^{-1} + iU\partial_\mu U^{-1} \qquad (6)$$

And also:

$$F\mu\nu = U\, F\mu\nu\, U^{\mu\nu} \qquad (7)$$

Curvature follows standard relativistic nature, and it works as usual in electromagnetic formalism.

3. CALIBRATION ACTION FUNCTIONAL ON U (1).

Using presented Lorenz invariants we write Maxwell calibration action [10]:

$$SM = \int \left(k1F\mu\nu F^{\mu\nu} + k2e^{\mu\nu\rho\sigma}F\mu\nu F\rho\sigma + k3A\mu J^\mu\right)d^4x \quad (8)$$

J^μ is quadric-current attached to field A_μ. Electric current and integral action along geometrical manifold it's represented by electric and magnetic fields.

4. FIELD EXPRESSIONS

Second term of action is a total quadric-divergence and it can be rewritten as follows:

$$S2 = \int k2\partial\mu(e^{\mu\nu\rho\sigma}A\mu\partial\rho A\sigma)d^4x \qquad (9)$$

One can rewrite S_M in terms of last equivalent expression:

$$SM = \int(k1F\mu\nu F^{\mu\nu} + k2\partial\mu(e^{\mu\nu\rho\sigma}A\mu\partial\rho A\sigma) + k3A\mu J^\mu)\, d^4x \qquad (10)$$

$$SM = \int(k1F\mu\nu F^{\mu\nu} + S2 + k3A\mu J^\mu)\, d^4x \qquad (11)$$

Analysis on S_2 integral show that this term it doesn't contribute to motion equations, and it is independent from regular dynamics of action, therefore one can assume in this case that $S_2 \rightarrow 0$, thus we will have the following expression:

$$SM = \int (k1F\mu\nu F^{\mu\nu} + k3A\mu J^{\mu})\, d^4x \tag{12}$$

Field variation lead us to the resulting field equations for calibration model. Solving last PDE, and integrating both terms from the right side of equation give us:

$$SM = \int (k1F\mu\nu F^{\mu\nu}) d^4x + \int (k3A\mu J^{\mu}) d^4x \tag{13}$$

$$SM = k1 \int F\mu\nu F^{\mu\nu} d^4x + k3 \int A\mu J^{\mu} d^4x \tag{14}$$

Then we obtain the following field equations for field A_μ Which is mathematical structure of Maxwell field theory in 4 dimensions.

$$k1\partial\mu F^{\mu\nu} = k3J^3 \tag{15}$$

Constants k_3 and k_3 might be interpreted depending on field lagrangian approximation. In many cases, Chern-Simmons lagrangian conception may result in topological order constants or states [4,10].

$$\delta L \alpha \frac{m}{4} \partial_\mu (e^{\mu\nu\rho} F_{\nu\rho}) \tag{16}$$

An interesting remark is that at low energy, the mass term dominates and this model with sources describes the integer Hall effect.

5. 2+1 CHERN-SIMMONS REDUCTION

One we have field equations for calibration theory on field A_μ we induce Chern-Simmons formalism by a 2+1 reduction [2,3]. This may be obtained as follows:

Considerate quadric-divergence term S_2 not equal to zero (S_2 6= 0) i.e:

$$S2 = \underline{\int} k2\partial\mu(e^{\mu\nu\rho\sigma} A\mu\partial\rho A\sigma)\, d^4x \neq 0 \tag{17}$$

And applying Gauss flux theorem, from Chern-Simmons electromagnetic formalism, it is possible to make a one-dimensional reduction:

$$4D \rightarrow 3D = 2 + 1dim. \tag{18}$$

For which Chern-Simmons integration and dimension 2+1 reduction is obtained from Gaussian integration since flux theorem:

$$SCS = \underline{\int} k2e^{\mu\nu\rho\sigma} A\nu\partial\rho A\sigma d^3x \tag{19}$$

Where S_{CS} is rewritten calibration action and it is known as Chern-Simmons expression for 2+1 dimensions. This new action may be used to obtain a new calibration theory due to electromagnetic formalism [2]. This is called Maxwell-Chern-Simmons approximation, and it will be written in the following section.

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6. MAXWEL AND CHERN-SIMMONS APPROXIMATION.

In this section Maxwell-Chern-Simmons approximation is constructed. From Maxwell and Chern-Simmons terms we obtain S_{MCS} action as follows:

$$SMCS = \int (k1F\nu\rho F^{\nu\rho} + k2e^{\mu\nu\rho\sigma}A\nu\partial\rho A\sigma + k3A\mu J^{\mu})\, d^3x \qquad (20)$$

Which is, as a matter of fact, the well-known Maxwell-Chern-Simmons action [12]. Solving integral equations for this action, one may manipulate terms, in order to obtain motion equations due to the next process:

$$S\,MCS = \int (k1F\nu\rho F^{\nu\rho})\, d^3x + \int (k2e^{\mu\nu\rho\sigma}A\nu\partial\rho A\sigma)\, d^3x + \int (k3A\mu J^{\mu})\, d^3x \qquad (21)$$

Due to the fact that tha k_1, k_2 and k_3 are constants, one may put it out from integrand S_{MCS1}, S_{MCS2} and S_{MCS3} respectively, which are the three terms of S_{MCS}. Thus, last integral may be rewritten as:

$$SMCS = k1\int (F\nu\rho F^{\nu\rho})\, d^3x + k2\int (e^{\mu\nu\rho\sigma}A\nu\partial\rho A\sigma)\, d^3x + k3\int (A\mu J^{\mu})\, d^3x \qquad (22)$$

Motion equations are given then by next expression:

$$\partial\rho F^{\nu\rho} - \frac{k2}{2k1}e^{\nu\rho\sigma}F\rho\sigma + \frac{k3}{k1}\frac{1}{4}J^{\nu} \qquad (23)$$

These field equations might be written in terms of Maxwell formalism in the next section. A new action can be written in terms of this formalism:

$$S = S_M + S_{CS} = \int d^3x \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - m\int d^3x \frac{1}{4}\epsilon^{\mu\nu\rho}A_{\mu}F_{\nu\rho} \qquad (24)$$

Finally, one can define lagrangian functional and its variation as:

$$\delta L \alpha \frac{m}{4}\partial_{\mu}(e^{\mu\nu\rho}F_{\nu\rho}) \qquad (25)$$

Which has expected form of equation 16.

7. CHERN-SIMMONS 2+1 ACTION IN MAXWELL FORMALISM.

Maxwell-Chern-Simmons action has been solved in the last section and obtained as result motion equations. One can rewrite motion equations in terms of Maxwell formalism, keeping invariant 2+1 Chern-Simmons calibration action dimensional reduction. Equations in terms of electrical and magnetic fields may be formulated as follows.

$$\nabla \circ \mathbf{E} - \frac{k_2}{k_1}\mathbf{B} = \frac{k_3}{4k_1}\rho, \qquad (26)$$

$$\partial_t \mathbf{E} + \nabla^* \mathbf{B} - (\frac{k_2}{k_1})\mathbf{E}^* = (\frac{k_3}{4k_1})\mathbf{J} \qquad (27)$$

$$\partial_t \mathbf{B} + \nabla^* \mathbf{E} = 0. \qquad (28)$$

In second Maxwell equation, E^* is the dual vector of electric field E. It is relevant to remark that magnetic field B in 2+1 dimensions is a pseudo-scalar field.

8. KLEIN-GORDON OUTPUT EQUATIONS.

As conclusions, one may conclude that it is possible to obtain calibration action for topological action given by Chern-Simmons theory. Topological action might be reduced to 2+1 dimensions if we vary its terms and make one of them zero (quadric-divergence term

Solving Maxwell equations, we obtain Klein-Gordon equations following mathematical development of [9] solutions are Klein Gordon equations with a Laplacian operator defined as:

$\square = d * d * + * d * d$, and equations are:

$$(-\square + (\frac{k_2}{k_1})^2)B \tag{29}$$

and:

$$(-\square + (\frac{k_2}{k_1})^2)E \tag{30}$$

We obtained these algebraic results in the moment when one makes zero k_3 and the distance among charge sources get far one from the other. These last two equations correspond to differential wave massive equations of Gordon-Klein's type. Equations (29) and (30) are equal to zero.

Chern-Simmons action and its formulation elements induced constants *k's* 1, 2 and 3 to be topological mass quantities, as well as we follow Chern-Simmons topological order nature. One can rewrite topological mass regarding to $\frac{k_2}{k_1}$ constant term. Therefore, we have the next topological mass expression:

$$m_t = (\frac{k_2}{k_1})^2 \tag{31}$$

Where m_t is considered by Chern-Simmons formalism as mass constant of topological order.

9. TOPOLOGICAL MASS ANALYSIS.

As one can see in [10] topological mass generation is a phenomenon in 2+1 dimensions discovered by the authors of [10]. These topologically nontrivial additions profoundly alter the particle content of the field models and lead us to quantization of a dimensionless mass-coupling-constant ratio. Although, gravitation field becomes scalar mass field, mediates finite-range interactions, and it has spin 2.

10. SUMMARY

Main ideas of this particular case presented in this article, besides other reviews about the same method to obtain generation of topological mass are gathered in the next 10 ideas:

1.- The Maxwell (or more generally Yang-Mills)-Chern Simons violates parity due to the antisymmetric tensor.
2.- Field equations describe a massive gauge field without being gauge noninvariant.
3.- Chern-Simmons formalism attached mass to field and potentials involved.
4.- Abelian symmetry on U(1) gives commutativity to Lorenz-Gauge invariance.
5.- It is possible to extend Chern-Simmons action to Maxwell formalism.
6.- Motion equations from Maxwell-Chern-Simmons action are solved as Klein Gordon equations.
7.- Topological mass is obtained from a resulting constant coefficient from Klein Gordon equations.
8.- It is possible to build a calibration action on u(1) Lie group.
9.- A 2+1 reduction is obtained while we make zero quadric-divergence term in Chern-Simmons action.
10.- Maxwell relativistic formalism is applied to represent Chern-Simmons calibration action.

11. CONCLUSIONS

As conclusions, one may conclude that it is possible to obtain calibration action for topological action given by Chern-Simmons theory. Topological action might be reduced to 2+1 dimensions if we vary its terms and make one of them zero (quadric-divergence term). We also may rewrite Chern-Simmons and expressed them in Maxwell relativistic formalism, solving those equations in order to obtain Klein-Gordon massive wave equations with topological mass interpretation. Authors interpretation is that it is possible to obtain topological mass each time when Chern-Simmons action is used over symmetry Lie groups, and expressed in terms of electromagnetic formalism, obtaining massive wave equations. Topological principles such as Chern-Simmons assumption of topological manifolds inherent to the model are responsible of resulting topological mass terms [13].

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